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The Flow of Very Dilute Polyox Solutions into a Region of Sudden Capillary Tube Enlargement

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Synopsis

The flow of dilute drag reducing polymer solutions through capillary tubes into a region of slightly greater tube diameter is characterized by almost complete recovery of the fluid. It is conjectured that the polymer molecules react to prevent the loss of kinetic energy by minimizing the formation of eddies resulting from the expansion.

INTRODUCTION

When a fluid passes into a section of sudden tube enlargement, the flow breaks away from the wall or separates, and eddying flow takes place. At some distance downstream, the flow will again follow the wall, but there will be a loss in head. Astarita and Nicodemo[1] have recently reported on the flow of aqueous vinyl polymer solutions into sudden enlargements, the pipe diameter in their experiments changing from 9.6 to 20.0 mm. They concluded that larger head losses would be associated with viscoelastic liquids than with Newtonian fluids. The head loss, h_0, is usually defined in the following way by most textbooks[2]:

$$h_0 = \left(\frac{p_1}{\rho g} + \frac{V_1^2}{2g}\right) - \left(\frac{p_2}{\rho g} - \frac{V_2^2}{2g}\right) \qquad (1)$$

where subscript 1 refers to the pipe section just before enlargement and subscript 2 refers to that section of the enlargement where the flow again follows the pipe wall.

As a supplement to capillary tube experiments with highly dilute Polyox solutions at this laboratory, a metal tube extension of slightly greater diameter was fitted to the main flow tube used. This report briefly explores the qualitative experimental results obtained through the use of Bernoulli's equation, pointing out the distinct differences in flow behavior observed between solvent and polymer solution in this particular flow system.

EXPERIMENTAL

The apparatus used essentially consisted of a motor-driven syringe attached to a Pyrex capillary tube. The apparatus was similar in principle to that used by Hoyt,[3] but liberal changes in design were instituted. Improvements included a flow rate measuring device—essentially a small d.c. generator—coupled to the motor drive unit. The apparatus design and operation are described in detail elsewhere.[4] Particulars of the flow tube used are as follows: diameter = 0.1575 cm, length between entrance and first tap = 31.50 cm, length between first and second tap = 15.75 cm, length between second tap and end of tube = 4.77 cm. length of extension tube = 3.65 cm, and diameter of extension tube = 0.1855 cm.

The Polyox coagulant sample used had a molecular weight of 7.0×10^6 based on intrinsic viscosity determinations[5] and the intrinsic viscosity–molecular weight relation reported by Shin.[6]

RESULTS AND DISCUSSION

Figure 1 contrasts the experimental flow behavior of 5.02 ppm Polyox coagulant with that of water both with and without the extension tube. Water, as expected, shows a decided pressure decrease due to the combined effects of added tube length and expansion losses. The Polyox solution, on the other hand, while showing decreased pressure drop at a given flow rate

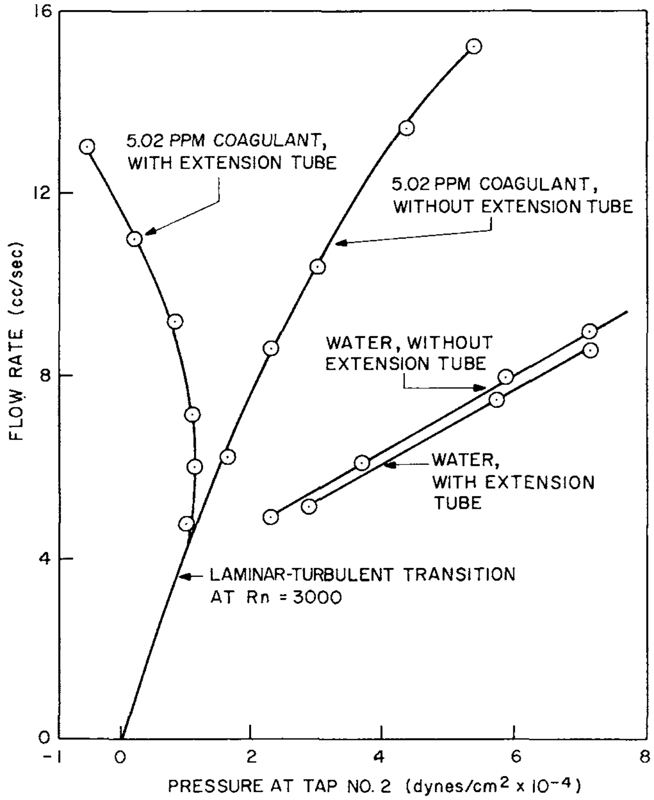

Fig. 1. Flow of Polyox coagulant and water into a region of sudden pipe enlargement.

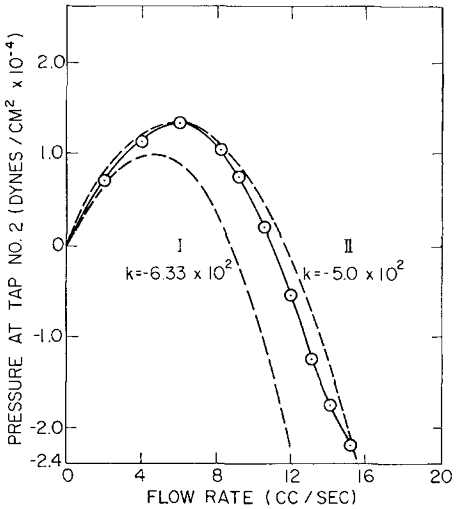

Fig. 2. Flow of 5.02 ppm Polyox coagulant compared with the calculated Venturi tube coefficient (I) and an empirical coefficient (II).

when compared with water for flows in the absence of the extension tube behaves quite differently when the extension tube is added to the main capillary tube. The pressure sensed at tap 2 (placed 4.77 cm from the joining face of the extension tube) becomes negative when sufficiently high flow rates are reached. The overall experimental effect appears qualitatively similar to what would have been expected had a highly efficient diffuser tube section been placed at the end of the main tube, with its "throat" positioned at the joining face of the tubes rather than the enlarged tube section. It will be of interest to determine to what extent and within what limits the flow into the enlarged section can be described by an idealized expansion of the drag-reducing fluid.

The pressure change caused by the expansion of fluid into an idealized diffuser tube section of length equivalent to that of the extension may be approximated as follows: From Bernoulli's equation,

$$P_v + {}^1/_2\rho V_v^2 = P_y + {}^1/_2\rho V_y^2, \tag{2}$$

then continuity

$$V_y = \frac{D_v^2 V_y}{D_y^2},$$

and assuming $P_y = 0$, one obtains

$$P_v = {}^1/_2\rho V_v^2\left[\left(\frac{D_v}{D_y}\right)^4 - 1\right]. \tag{3}$$

Since $V_v = \dfrac{4Q}{\pi D_v^2}$ and $\rho \cong 1$,

$$P_v = \frac{8}{\pi^2}\left(\frac{1}{D_y^4} - \frac{1}{D_v^4}\right)Q^2. \tag{4}$$

(For the present system, D_v and D_y are known, and eq. (4) reduces to $P_v = -6.33 \times 10^2 Q^2$.)

The pressure drop between the diffuser "throat" and the pressure tap is known from flow data collected between the pressure taps in the main tube. The pressure drop due to friction in the extension tube can be estimated by assuming that the flow resistance is similar to that of the main tube by applying a factor equal to the fourth power of the diameter ratio. The estimated relation for the pressure which should be observed at pressure tap number 2 would be

$$P_T = L_B \nabla P + \left[L_E \times \left(\frac{D_v}{D_y} \right)^4 \right] \nabla P + \frac{8}{\pi^2} \left(\frac{1}{D_y^4} - \frac{1}{D_v^4} \right) Q^2 \qquad (5)$$

Figure 2 shows the application of this standard analysis to the flow data. The dashed curves represent two cases. Curve I represents the use of the the computed coefficient for the term involving the squared flow rate based on the tube diameters given, i.e., $K = -6.33 \times 10^2$; curve II makes use of a loss coefficient designed to yield a maximum corresponding to the one observed in the given experimental flow curve. In all cases, the flow curve generated by use of a coefficient based on the given tube diameters fell slightly below the experimental flow curve. The use of the loss coefficient indicative of a less efficient flow expansion yielded better agreement over the observable flow rate range. However, the agreement with the idealized diffuser tube model is perhaps as good as could be expected based on the simple model considered here and in the absence of a sure knowledge of the flow in the extension tube.

The major point made by the capillary tube data is that almost complete recovery is obtained following the expansion of the polymer solution into the enlargement. It would appear that the polymer molecules react to prevent the loss of kinetic energy perhaps by minimizing the formation of eddies resulting from the expansion. That is, in contrast to the water data of Figure 1, most of the kinetic energy of the polymer solution appears to be converted into pressure energy, as illustrated in Figure 2. However, in the light of Astarita and Nicodemo's work,[1] the effect would appear to be confined to pseudolaminar flows of dilute drag-reducing polymer solutions (having negligible elasticity) into regions of slight capillary enlargement.

Nomenclature

L_B	tube length between pressure tap and joining face of tubes
L_E	length of extension tube
∇P	pressure gradient in main tube
P_v	pressure developed at joining face of tubes in absence of frictional losses
P_y	pressure at terminus of extension tube
Q	flow rate
V	mean velocity of the flow
V_v	mean velocity in main tube

V_y	mean velocity in extension tube
ρ	density of fluid
Re	Reynolds number
K	empirical constant
g	gravitational constant

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